**ASSIGNMENT**

**EVEN(GROUP-2)**

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| **220626** | **220628** | **220632** | **220634** | **220636** |
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**Question-1.Explain Conditions for the Existence of the Fourier Transform.**

The Fourier Transform exists for a function *x(t)* if it satisfies certain conditions known as the Dirichlet Conditions. These conditions ensure that the function can be represented in terms of sinusoidal components. The conditions are:

### **1. Absolute Integrability (Energy Constraint)**

The function *x(t)* must be absolutely integrable over the entire time domain:

This condition ensures that the total energy of the function is finite, which is necessary for the Fourier transform to converge.

### **2. Finite Number of Discontinuities**

The function *x(t)*must have a finite number of discontinuities in any finite interval. If there are infinite discontinuities, the Fourier transform may not exist or may not converge properly.

### **3. Finite Number of Maxima and Minima**

In any finite interval, *x(t)*should have a finite number of maxima and minima. This ensures that the function does not oscillate too wildly, allowing the Fourier transform to be well-defined.

### **4. Bounded Growth**

The function should not grow too fast. More formally, *x(t)* should be of exponential order, meaning there exist constants *M* and *a* such that:

*∣x(t)∣≤M*

for sufficiently large *|t|*. This ensures that the function does not increase too rapidly, which would make the Fourier transform diverge.

### **Conclusion**

If a function satisfies these conditions, its Fourier Transform exists and can be computed using:

*dt*

If any of these conditions are violated, the Fourier transform may not exist in the conventional sense, but sometimes it can be defined in a generalized sense (e.g., using distributions like the Dirac delta function).

**Question-2:Why does DFT produce both positive and negative frequencies?**

The Discrete Fourier Transform (DFT) produces both positive and negative frequencies due to its fundamental mathematical nature, which stems from the way complex exponentials are used to represent signals. Here’s a deeper dive into why this happens:

**1. DFT and Complex Exponentials**

The DFT decomposes a signal into sinusoids of different frequencies. These sinusoids are represented as complex exponentials of the form:

where:

* j is the imaginary unit (),
* k is the frequency index (which can be positive or negative),
* n is the sample index,
* N is the total number of samples.

The expression is a complex sinusoid, which is a combination of a cosine and sine wave due to Euler's formula:

= cosθ +j sinθ

Thus, each frequency component in the DFT is a complex exponential, which inherently has both real and imaginary parts. The positive and negative frequencies correspond to the direction of rotation of these complex exponentials in the complex plane.

**2. Symmetry of the DFT**

When you compute the DFT, you get frequency components for both positive and negative values of kk. These are not two different types of frequencies but rather represent different directions of rotation in the complex plane:

* **Positive frequencies** correspond to counterclockwise rotations (increasing in frequency).
* **Negative frequencies** correspond to clockwise rotations (decreasing in frequency).

In a real-valued signal (the most common case), the DFT exhibits symmetry. Specifically:

**X[k]=X\*[N−k]X[k]**

where X[k] is the DFT at index k, X\* is the complex conjugate, and N is the total number of points in the DFT. This symmetry means that the negative frequencies contain redundant information and are complex conjugates of the positive frequencies. They don’t represent separate physical frequencies but are part of the overall signal's Fourier representation, ensuring the result is consistent with the real-valued nature of the input signal.

**3. Interpretation of Negative Frequencies**

The concept of negative frequencies might seem abstract at first, but in signal processing, they arise naturally when analyzing signals using complex exponentials.

* **Positive frequencies** represent sinusoids that oscillate in the counterclockwise direction in the complex plane.
* **Negative frequencies** represent sinusoids that oscillate in the clockwise direction.

In the context of real-valued signals, the negative frequencies are essentially a mirror image of the positive frequencies in terms of their contribution to the signal, due to the symmetry of the Fourier transform.

**4. Physical Meaning**

When we look at a real-valued signal in the time domain (like an audio signal or a waveform), it's composed of components that have both positive and negative frequency components due to the way sine and cosine functions combine. For example:

* A **pure tone** (sinusoidal wave) with frequency ff can be expressed as the sum of two complex exponentials: one with a positive frequency ff and the other with a negative frequency −f-f.
* In practice, you only observe the real part of these complex exponentials, but the negative frequency components are still crucial in constructing the signal.

Thus, negative frequencies don't represent distinct physical components but rather are a mathematical artifact of using complex exponentials to represent real-world signals.

**5. Why Both Positive and Negative Frequencies?**

The need for both positive and negative frequencies arises from the requirement of representing both the magnitude and phase of the oscillatory components of the signal. A signal like a cosine wave, which might oscillate in one direction, requires both the positive and negative frequencies to fully capture the phase and amplitude information of its sinusoidal components.

**Summary**

The DFT produces both positive and negative frequencies because:

* It represents the signal as a sum of complex exponentials, which inherently include both directions of oscillation.
* Negative frequencies are just the complex conjugates of positive frequencies and are necessary for correctly representing real-valued signals.
* This dual representation allows for a complete reconstruction of the signal, preserving both magnitude and phase information, even though, in practice, negative frequencies do not correspond to distinct physical phenomena but are part of the mathematical formulation of the transform.

**Question-3.What is the sampling theorem? Derive the Nyquist rate.**

**The Sampling Theorem**

The Sampling Theorem (also known as the Nyquist-Shannon Sampling Theorem) is a fundamental concept in signal processing, particularly in the context of converting continuous-time signals to discrete-time signals. It defines the conditions under which a continuous-time signal can be perfectly represented by its samples.

### **Statement of the Sampling Theorem:**

A continuous-time signal x(t) can be completely reconstructed from its samples, taken at regular intervals, if and only if the sampling rate fs (also called the sampling frequency) is greater than twice the highest frequency present in the signal. Mathematically, this is expressed as:

​≥2​

where:

* is the sampling frequency (samples per second or Hz),
* is the highest frequency component in the signal x(t).

### **Important Concepts:**

1. **Sampling**: This refers to the process of taking discrete samples of a continuous-time signal x(t) at uniform time intervals Ts=, where fs​ is the sampling frequency.
2. **Nyquist Rate**: The Nyquist rate is the minimum sampling rate required to avoid aliasing, which occurs when fs is less than 2​. The Nyquist rate is simply 2​.
3. **Aliasing**: If the sampling frequency is lower than the Nyquist rate, aliasing occurs. This means higher frequency components in the signal fold back into the lower frequency range, causing distortion in the reconstructed signal. Aliasing is undesirable because it makes the signal indistinguishable from other frequencies.
4. **Perfect Reconstruction**: When the sampling rate satisfies the Nyquist criterion, the original signal can be perfectly reconstructed from the sampled signal using an interpolation process (such as the sinc function interpolation).

### Example:

Consider a continuous-time signal x(t)=sin(2π t)

Where fmax​ is the maximum frequency component in the signal.

* To sample x(t) without aliasing, the sampling frequency fs must be greater than 2.
* If the signal has a maximum frequency of 5 kHz (i.e., ​=5000 Hz), the minimum sampling rate required is =2⋅5000=10000 Hz (10 kHz).
* Sampling below 10 kHz would lead to aliasing.

### Need for Sampling theorem:

The Sampling Theorem ensures that continuous signals can be captured and reconstructed without loss of information, as long as the signal is sampled at a rate that is at least twice the maximum frequency component present in the signal. If this condition is not met, aliasing will distort the signal. The Sampling Theorem is crucial in the field of signal processing because it provides the foundation for converting continuous-time signals (analog signals) into discrete-time signals (digital signals) while ensuring that no information is lost during the conversion.

# The Nyquist rate is the minimum sampling rate required to avoid aliasing when converting a continuous-time signal into a discrete-time signal.

# Derivation of Nyquist Rate

Let x(t) be a band-limited signal with a maximum frequency component , meaning its Fourier transform satisfies:

X(f) = 0, |f| >

Sampling x(t) at a rate = produces a discrete-time signal:

Taking the Fourier transform of (t), we obtain its spectrum as:

This equation shows that the original spectrum X (f) repeats at integer multiples of . For perfect reconstruction, these shifted spectra should not overlap, leading to the condition:

Thus, the minimum sampling rate required to avoid aliasing is:

which is known as the Nyquist rate.

**Question-4: Define stability in terms of system response**.

A system is said to be stable if its output remains bounded for any bounded input. This means that the system does not produce an infinite or unmanageable response when subjected to a finite input.

**Bounded Input - Bounded Output (BIBO) Stability:**

A system is BIBO stable if, for every bounded input x(t) or x[n], the output y(t) or y[n] remains bounded.

Mathematically, if there exists a constant A such that:

∣x(t)∣ ≤ A < ∞ for all t

then the system is stable if there exists another constant BBB such that:

∣y(t)∣ ≤B <∞ for all t.

**Mathematical Condition for Stability:**

1. For Continuous-Time LTI Systems:

The system is stable if its impulse response h(t) satisfies:

2. For Discrete-Time LTI Systems:

The system is stable if its impulse response h[n] satisfies:

If these conditions hold, the system is stable; otherwise, it is unstable.

**Example of a Stable System**

Consider the causal exponential system with impulse response:

h(t)=e(^−t) \* u(t),t≥0

where u(t) is the unit step function.

#### **Checking Stability Condition (Continuous-Time):**

Since the integral is finite, the system is BIBO stable.

### **Example of an Unstable System**

Consider the growing exponential system with impulse response:

h(t)=e^(t) \*u(t) ,t≥0

#### **Checking Stability Condition (Continuous-Time):**

e^t dt=∞

Since the integral diverges (goes to infinity), the system is unstable.

**Question-5: Properties of Fourier Transform.**

### **Properties of the Fourier Transform**

The Fourier transform is a powerful mathematical tool used to analyze signals and systems in both continuous and discrete domains. It transforms a signal from the time domain (or spatial domain) into the frequency domain, revealing its frequency components. The Fourier transform has several key properties that make it easier to work with and interpret. Below are some of the most important properties:

### **1. Linearity**

The Fourier transform is a linear operation. This means that if you have two signals and

, and two constants a and *b*, the Fourier transform of their linear combination is the same as the linear combination of their Fourier transforms.

F{*a*⋅*x*(*t*)+*b*⋅*y*(*t*)}=*a*⋅*X*(*f*)+*b*⋅*Y*(*f*)

* **Implication**: This property allows us to break down complex signals into simpler components and analyze them separately.

### **2. Time Shifting**

If a signal x(t) is shifted in time by ​, its Fourier transform is multiplied by a complex exponential.

F{*x*(−)}=*X*(*f*)⋅

* **Implication**: Time shifting does not change the magnitude of the Fourier transform but introduces a phase shift proportional to the frequency and the time delay.

### **3. Frequency Shifting (Modulation)**

If a signal x(t) is multiplied by a complex exponential , its Fourier transform is shifted in frequency by .

F{*x*(*t*)⋅}=*X*(-)

* **Implication**: This property is the basis for modulation techniques in communication systems, where signals are shifted to different frequency bands for transmission.

### **4. Time Scaling**

If a signal x(t) is scaled in time by a factor *a*, its Fourier transform is scaled inversely in frequency.

F{*x*(*at*)}=*X*()

* **Implication**: Compressing a signal in time (*a*>1 ) expands its frequency spectrum, and vice versa.

### **5. Duality**

The Fourier transform exhibits a duality between the time and frequency domains. If *X*(*f*) is the Fourier transform of x(t), then:

F{*X*(*t*)}=*x*(−*f*)

* **Implication**: This property allows us to derive new Fourier transform pairs by swapping the roles of time and frequency.

### **6. Convolution**

The Fourier transform of the convolution of two signals *x*(*t*) and *y*(*t*) is the product of their Fourier transforms.

F{*x*(*t*)∗*y*(*t*)}=*X*(*f*)⋅*Y*(*f*)

* **Implication**: Convolution in the time domain corresponds to multiplication in the frequency domain, simplifying the analysis of linear systems.

### **7. Multiplication**

The Fourier transform of the product of two signals *x*(*t*) and *y*(*t*) is the convolution of their Fourier transforms.

F{*x*(*t*)⋅*y*(*t*)}=*X*(*f*)∗*Y*(*f*)

* **Implication**: Multiplication in the time domain corresponds to convolution in the frequency domain, which is useful in modulation and filtering.

### **8. Differentiation**

The Fourier transform of the derivative of a signal *x*(*t*) is proportional to the Fourier transform of the original signal multiplied by *j*2*πf*.

F{}=

* **Implication**: Integration in the time domain attenuates high-frequency components, as the factor decreases with frequency.

### **10. Parseval's Theorem**

Parseval's theorem states that the total energy of a signal in the time domain is equal to the total energy in the frequency domain.

dt = df

* **Implication**: This property ensures that the Fourier transform preserves energy, making it useful for power and energy calculations.

### **11. Symmetry**

For real-valued signals *x*(*t*), the Fourier transform *X*(*f*) exhibits Hermitian symmetry:

*X*(−*f*)=*X*∗(*f*)

* **Implication**: The magnitude spectrum is even, and the phase spectrum is odd, which simplifies analysis for real signals.

### **12. Time Reversal**

If a signal *x*(*t*) is reversed in time, its Fourier transform is also reversed in frequency.

F{x(−t)}=X(−f)

* **Implication**: Time reversal corresponds to frequency reversal, which is useful in certain signal processing applications.